An Improved Technique for Reducing Baseband Tones in Sigma–Delta Modulators Employing Data Weighted Averaging Algorithm Without Adding Dither

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***Abstract*—The data weighted averaging (DWA) algorithm used in multi-bit sigma–delta modulators (SDM) is troubled by baseband tones resulting from component mismatch of the SDM's internal multi-bit digital-to-analog converter (DAC). In this paper, we analyze DAC baseband tones and find them closely correlated to the number of unit elements used in the DAC. An improved technique is proposed for shifting the DAC tones away from the baseband without adding dither. For third-order sigma–delta modulators with an oversampling ratio (OSR) of 64 and either a nine-level or eight-level internal DAC with 0.5%–2% random component mismatches, simulation reveals that the DWA algorithm with the proposed technique can achieve nearly perfect first-order DAC noise shaping in the baseband and, on the average, 12 dB improved signal-to-(noise and distortion) ratio and 20 dB improved in-band distortion.**

I. Introduction

A major obstacle for superior multi-bit sigma–delta modulators (SDM's) is that good linearity of the SDM's internal multi-bit digital-to-analog converter (DAC) requires high component matching [1]. Good attenuation of DAC noise due to component mismatch is provided by the data-weighted averaging (DWA) algorithm (hereafter called DWA) [2], which ideally can achieve first-order DAC noise shaping [3], [4]. DWA has been successfully applied in very high-resolution (19-bit) sigma–delta analog-to-digital converters (ADC's) for dc measurements [5]. However, DWA can cause the aliasing of DAC tones into the baseband, resulting in reduction of SDM performance [2]. DWA aliasing tones in a multi-bit SDM can be broken up and randomized by adding dither, at the cost of increasing baseband noise, reducing dynamic range, and possibly destabilizing the modulator [2].

In this paper, DWA aliasing in a multi-bit SDM is analyzed. A new technique for reducing DWA baseband aliasing tones is proposed which, unlike adding dither, induces no performance degradation. Moreover, with the proposed technique, DWA delivers nearly perfect first-order DAC noise shaping, with no notable baseband aliasing tones. Performance improvements for the proposed technique are analyzed using Monte Carlo simulations and demonstrated by third-order modulators with an oversampling ratio (OSR) of 64, incorporating either a nine-level or eight-level internal DAC, with random component mismatches from 0.5% to 2%.

II. Data Weighted Averaging Algorithm Employed in Multi-Bit DAC's

DWA cyclically selects unit elements used in multi-bit D/A conversions. Fig. 1(a) demonstrates the principle for a nine-level eight-unit-element DAC implemented by switched-capacitor circuits. In an SDM, DAC feedback is built around the SDM integrator stage as shown in Fig. 1(b). Hence, DAC input codes, i.e., SDM output codes

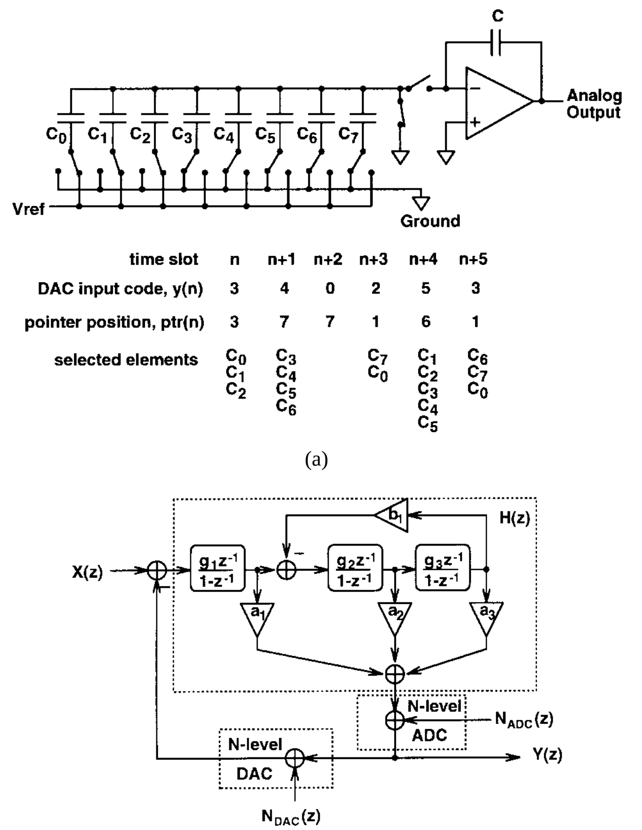

Fig. 1. (a) Principle of the data-weighted averaging (DWA) algorithm. (b) Linear model of a third-order multi-bit sigma–delta modulator.

denoted as $y(n)$, range from zero to eight. As shown in Fig. 1(a), $\mathrm{ptr}(n)$ is the pointer position which addresses the first of the elements to be selected at time $(n+1)$. As defined in [5], $\mathrm{IM}(\mathrm{ptr}(n))$, referred to as integral mismatch, is the accumulation of the element mismatch error from position $i=0$ to $i=(\mathrm{ptr}(n)-1)$ and can be expressed as $\mathrm{IM}(\mathrm{ptr}(n)) = \sum_{i=0}^{\mathrm{ptr}(n)-1}(C_i - C_{\mathrm{mean}})/(C_{\mathrm{mean}}) = \sum_{i=0}^{\mathrm{ptr}(n)-1} e_i$ where C_{mean} is the average value of the unit capacitors, $C_{\mathrm{mean}} = 1/N\sum_{i=0}^{N-1} C_i$, C_i is the ith capacitor value, e_i is the mismatch error of ith unit capacitor, and N is the number of total unit capacitors. Mathematically, in the DWA algorithm, DAC noise $N_{\mathrm{DAC}}(z)$ can be shown to be a function of the $\mathrm{IM}(\mathrm{ptr}(n))$ in Z-domain [5]

$$N_{\mathrm{DAC}}(z) = \left(1 - z^{-1}\right)\mathrm{IM}(\mathrm{PTR}(z)) \tag{1}$$

where $(1 - z^{-1})$ is a first-order noise shaping function. Nys and Henderson [3], [4] further derived two extreme cases for the integral mismatch function $\mathrm{IM}(\mathrm{ptr})$. For the first case, when the DAC input codes from one to N have equal probability $1/N$ at each clock cycle, $\mathrm{IM}(\mathrm{ptr})$ has a white spectrum. For the second case, when the consecutive DAC input codes have the same value u, the frequency spectrum of $\mathrm{IM}(\mathrm{ptr})$ is not white but rather is composed of discrete components at the frequencies

$$f_{\mathrm{tone}} = \frac{r}{N}\cdot f_s \cdot m, \qquad m = 1, 2, 3\cdots \tag{2}$$

where r is the value of the greatest common denominator (g.c.d.) of the DAC input code u and the number of total unit elements N, and f_s is the sampling frequency of the DAC. For example, when

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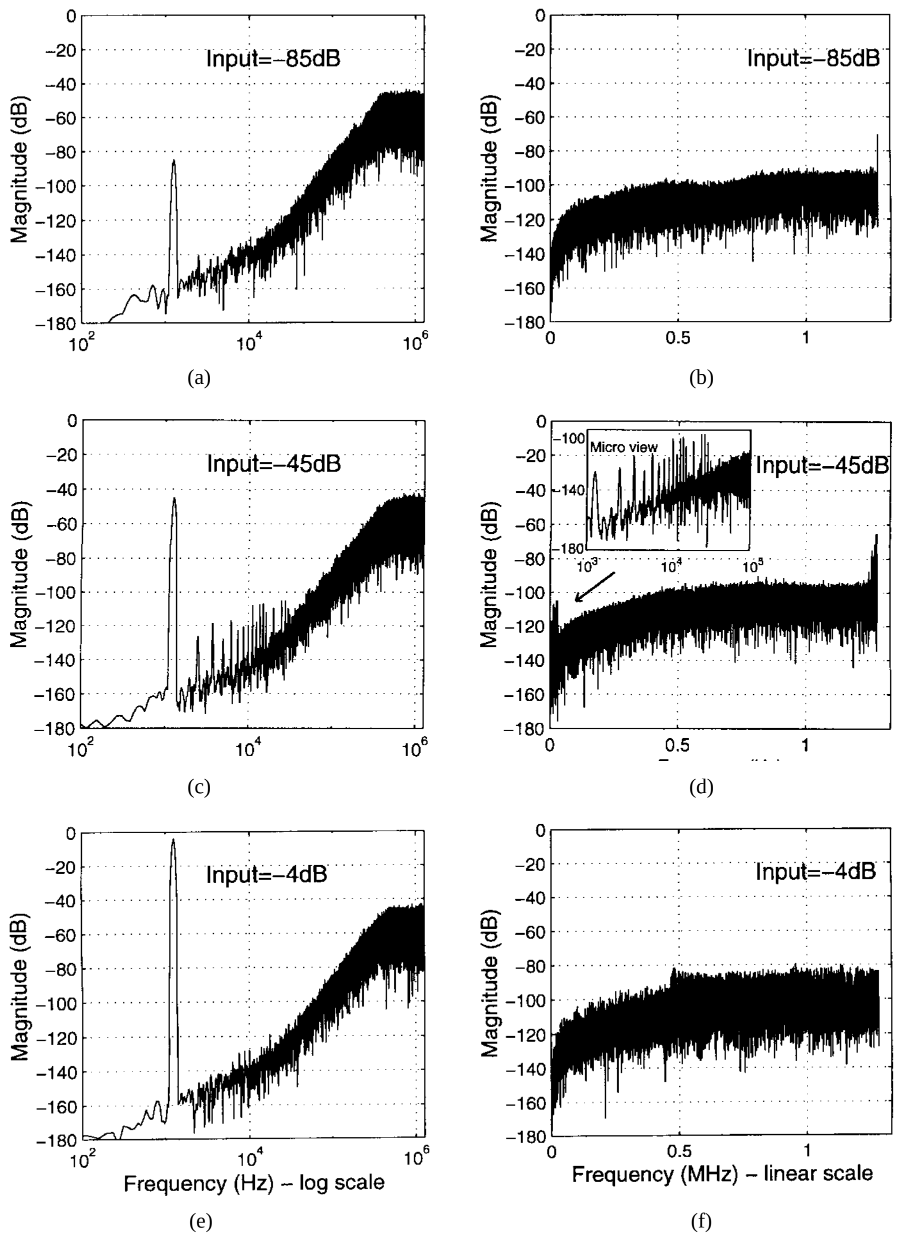

Fig. 2. Output spectra of SDM employing DWA algorithm with the input magnitudes of (a) −85 dB, (c) −45 dB, and (e) −4 dB. The corresponding DAC noise spectra are shown in (b), (d), and (f). In (a), (c), and (e), the unit of X-axis is hertz in log scale. In (b), (d), and (f), the unit of X-axis is megahertz in linear scale.

the consecutive DAC input codes have the same dc value of two for the nine-level eight-element DAC in Fig. 1(a), DAC noise due to element mismatch is a periodic sequence of e_0+e_1, e_2+e_3, e_4+e_5, and e_6+e_7, with discrete frequencies at $(f_s/4)\cdot m$ where m are positive integers.

DWA applied to an internal multi-bit DAC of a multi-bit SDM, which generates aliasing baseband tones, will be described in the following section.

III. Aliasing of Sigma–Delta Modulators with the DWA Algorithm

A third-order multi-bit SDM used as an example in this paper can be modeled as a linear system with separate additive quantization noise $N_{\rm ADC}(z)$ and DAC noise $N_{\rm DAC}(z)$, as shown in Fig. 1(b). Based on the linear model, the output $Y(z)$, composed of input signal $X(z)$, quantization noise $N_{\rm ADC}(z)$, and DAC noise $N_{\rm DAC}(z)$, can be expressed as $Y(z)=(H(z))/(1+H(z))X(z)+1/(1+H(z))N_{\rm ADC}(z)+(H(z))/(1+H(z))N_{\rm DAC}(z)$. Clearly, DAC noise cannot be shaped by the loop filter $H(z)$ and can be regarded as input signal.

For a multi-bit SDM, even with an ideal internal multi-bit DAC, notable tones in the baseband can be generated due to limit cycle oscillations of sigma–delta modulation, particularly for low-order modulators. Thus, for our analysis of DWA-induced tones, notable baseband tones due to sigma–delta modulation must be excluded. The modulator coefficients of the third-order SDM shown in Fig. 1(b) are obtained using the design methodology in [6] for generating low SDM baseband tones and $a_1=0.649\,35$, $a_2=0.416\,67$, $a_3=0.621\,76$, $g_1=2.930\,18$, $g_2=1.195\,95$, $g_3=0.196\,86$, and $b_1=0.007\,67$. The resulting SDM with an OSR of 64 has a 121-dB peak signal-to-(noise and distortion) ratio (SNDR), a 123-dB dynamic range, and a −3 dB maximum stable input magnitude. This SDM, with 20-kHz bandwidth and 2.56-MHz sampling frequency, is used as an example throughout this paper. In addition, this SDM with an ideal 3-bit (i.e.,

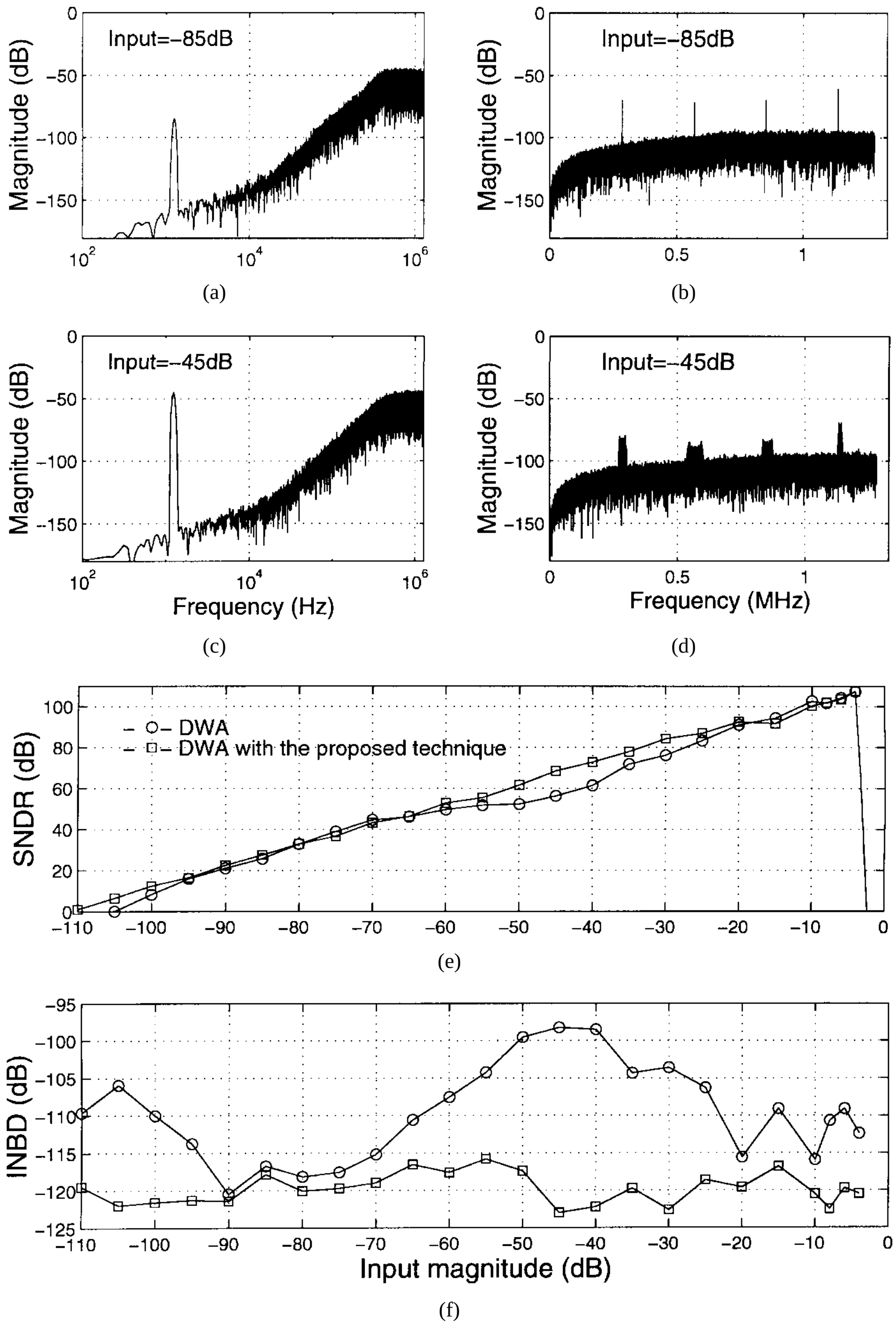

Fig. 3. Plots of (a) SDM output spectrum and (b) DAC noise spectrum with an input magnitude of −85 dB, (c) SDM output spectrum, and (d) DAC noise spectrum with an input magnitude of −45 dB, and (e) SNDR and (f) INBD for the DWA algorithm with the proposed technique where the DAC has nine levels and the input frequency is $f_s/2048$.

eight-level) internal DAC, shows no obvious baseband tones in the modulator output. Hence, it can be assumed that any baseband tones in the SDM output, with a nonideal DAC, are generated by DAC noise due to element mismatch.

For a multi-bit SDM, the DAC inputs contain quantization noise, DAC noise, and the input signal. When DWA is employed in the SDM, the mathematical derivation of DAC noise with such inputs is very complicated. Monte Carlo simulation bypasses this problem and is a useful tool for investigating DAC noise in an SDM employing DWA. In the following, the analyses of DAC noise are divided into multi-bit SDM's with odd and even quantization levels. The aforementioned SDM in two configurations, one with nine quantization levels and the other with eight, is used for demonstrative examples. In conventional design, a nine-level (eight-level) DAC requires eight (seven) unit elements.

The aforementioned SDM with a nine-level eight-element DAC whose maximum component mismatch is 0.5%, the SDM output and DAC noise spectra with SDM input magnitudes of −85, −45, and −4 dB, and an input frequency of $f_s/2048$ are illustrated for comparison and explanation. Component mismatch errors used for simulation in this example, obtained from a random number generator, are $e_0 = -0.005\,017\,79$, $e_1 = 0.001\,997\,9$, $e_2 = -0.001\,109\,4$, $e_3 = 0.004\,352\,4$, $e_4 = 0.001\,056\,0$, $e_5 = -0.001\,379\,16$, $e_6 = -0.004\,917\,79$, and $e_7 = 0.005\,017\,8$. When the SDM input magnitude is very small, the largest portion of the DAC input codes, i.e., the SDM output codes, are almost exclusively concentrated at the middle

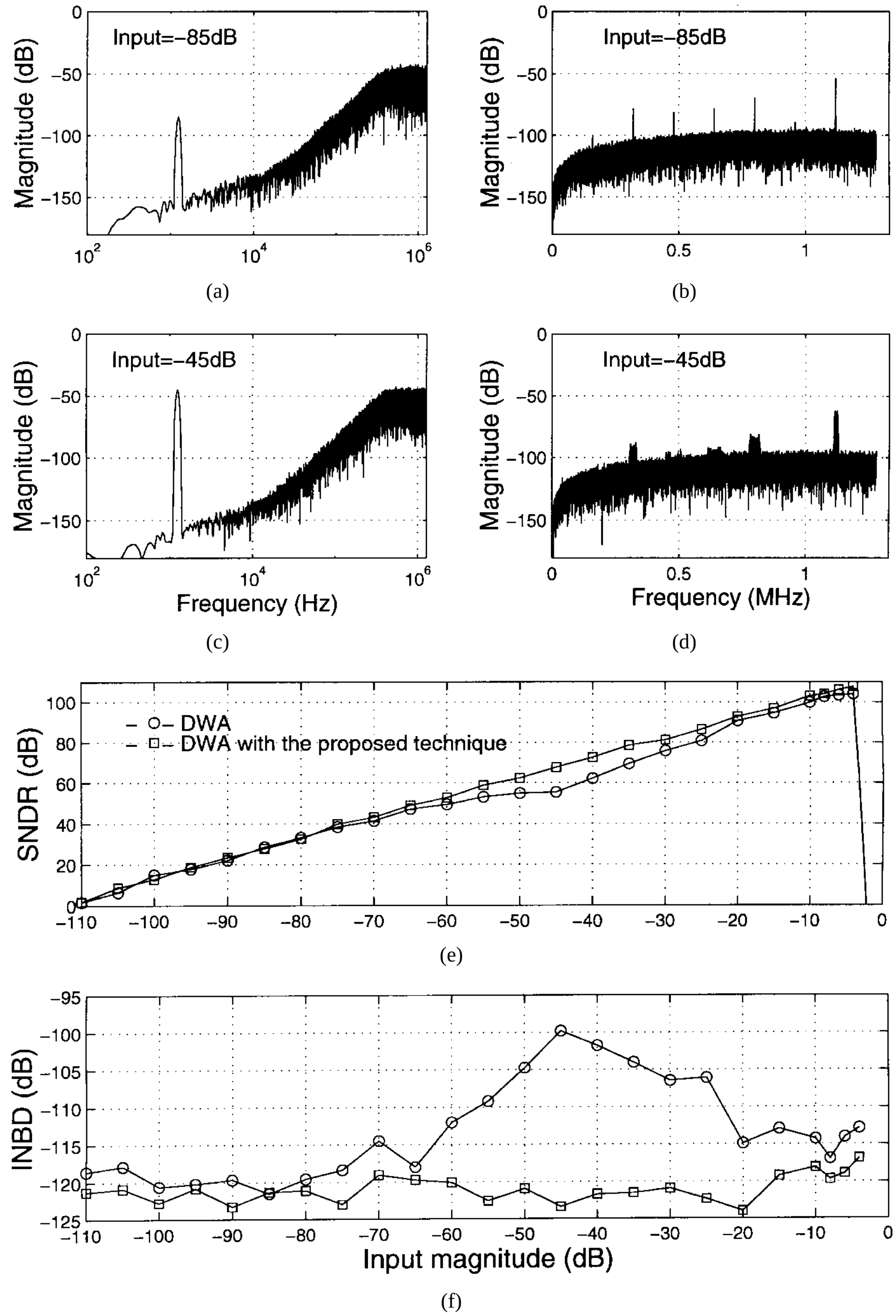

Fig. 4. Plots of (a) SDM output spectrum and (b) DAC noise spectrum with an input magnitude of -85 dB, (c) SDM output spectrum, and (d) DAC noise spectrum with an input magnitude of -45 dB, and (e) SNDR and (f) INBD for the DWA algorithm with the proposed technique where the DAC has eight levels and the input frequency is $f_s/2048$.

of a full scale of the SDM's internal DAC. For example, with a -85 dB SDM input magnitude, codes 3, 4 and 5 occupy 21%, 58%, and 21% of the DAC input codes, respectively. The distribution of DAC input codes is very nonuniform. Because the probability of code 4 is much larger than the others, this resembles inputting consecutive dc codes 4 into the DAC, and the notable tone frequency $(f_s/2) \cdot m$ will appear as shown in Fig. 2(b) where the first-order DAC noise shaping curve can also be observed. Because the tone level is lower than the SDM quantization noise level, no obvious tones, aliased from the DAC tones, are observed in the SDM output spectrum of Fig. 2(a). When the SDM input magnitude is large, the input of the SDM's internal ADC has a wider range of variations within the full scale. For an SDM input magnitude of -4 dB, the DAC input code probabilities, from code 1 to code 7, are 9.5%, 18.2%, 15.2%, 13.2%, 15.1%, 18.4%, and 9.4%, respectively. Simulation indicates that the DAC input code distribution is approximately uniform, i.e., each code has an equal probability. Therefore, DAC noise can be regarded as a white spectrum with first-order noise shaping, and no obvious tones are observed in the baseband, as shown in Fig. 2(e) and (f). However, with SDM input ranging between -60 dB and -30 dB, notable DAC tones are observed in two bands, one centered at dc and the other centered at $f_s/2$. As a result, many of the DAC tones are aliased to the SDM baseband. For example, at -45 dB SDM input, codes 3, 4, and 5 occupy 24%, 52%, and 24% of the DAC input codes, respectively. Substantial tones in the baseband of the SDM output, shown in Fig. 2(c), are aliased from DAC tones, as shown in Fig. 2(d). These

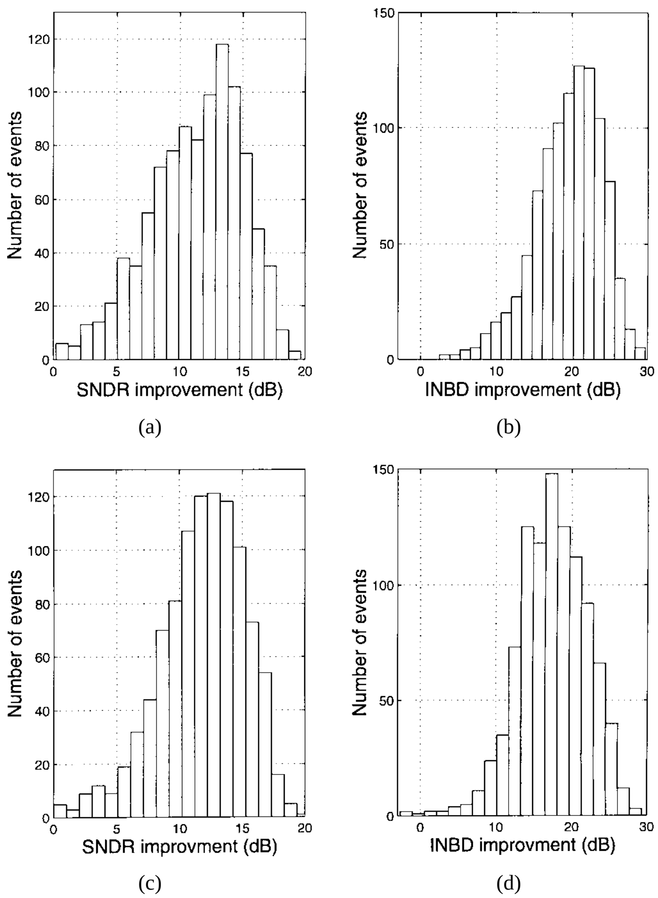

Fig. 5. Histograms of (a) SNDR and (b) INBD improvements for the DAC with nine levels, and (c) SNDR and (d) INBD improvements for the DAC with eight levels where the input magnitude is −45 dB and maximum component mismatch is 0.5%.

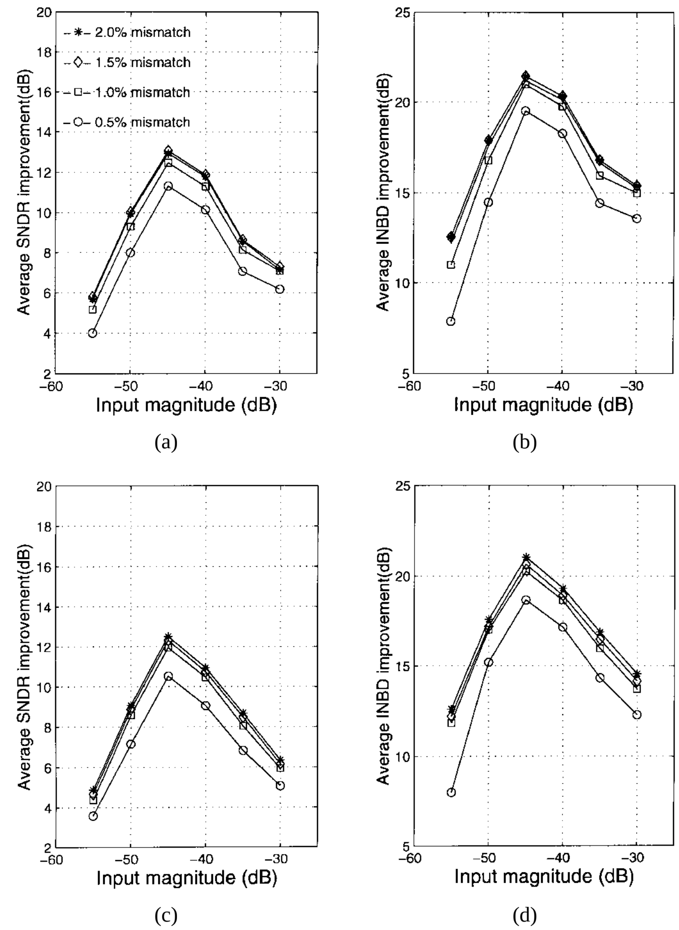

Fig. 6. Plots of average (a) SNDR and (b) INBD improvements for the DAC with nine levels, and (c) SNDR and (d) INBD improvements for the DAC with eight levels.

tones reach 25 dB above the noise floor in the baseband. Further, the aliased tones outside the modulator output baseband are not obvious since their levels are below the SDM quantization noise floor.

For the SDM with an eight-level seven-element DAC whose maximum component mismatch is 0.5% ($e_0 = -0.004\,346\,86$, $e_1 = -0.001\,248\,55$, $e_2 = -0.001\,264\,77$, $e_3 = -0.000\,159\,78$, $e_4 = 0.003\,904\,8$, $e_5 = 0.004\,694\,6$, and $e_6 = -0.001\,579\,39$), the SDM output and the DAC noise spectra for SDM input magnitudes of −85 dB, −45 dB, and −4 dB are very similar to Fig. 2, but the tone generation behavior is a little different from that of the nine-level SDM. With a very small SDM input magnitude, DAC input codes are concentrated at the middle of the full scale. With a −85-dB SDM input, codes 2, 3, 4, and 5 occupy 3%, 47%, 47%, and 3% of the DAC input codes, respectively. The high probability of codes 3 and 4, which are interleaved, makes notable tones at $(f_s/2) \cdot m$. For the eight-level DAC incorporating seven unit elements, DAC input sequence codes 3, 4, 3, 4, ... , produce DAC noise as a periodic sequence, $e_0 + e_1 + e_2$, $e_3 + e_4 + e_5 + e_6$, $e_0 + e_1 + e_2$, $e_3 + e_4 + e_5 + e_6, \cdots$, resulting in the $(f_s/2) \cdot m$ tone.

IV. Improved Technique for the DWA Algorithm

For DWA to be more useful in multi-bit SDM's, the DAC baseband tones must be removed. Conventionally, the problem is circumvented by adding dither. However, adding dither contributes additional noise to the baseband, degrades SNR, and possibly destabilizes the modulator [2]. This paper presents a new solution requiring no dithers as follows.

A. SDM's with Odd Quantization Levels

For an SDM with an $(N+1)$-level N-element DAC where $(N+1)$ is an odd number, notable baseband tones will occur because DAC input codes will mostly be code $N/2$ at small SDM input magnitudes. The proposed technique reduces SDM baseband tones by the addition of k extra unit elements to the selection elements of the SDM internal DAC, thereby moving notable tones out of the baseband, with no change to the quantization levels of the SDM's internal ADC and DAC. Inputting dc codes to the DAC with k extra elements, DAC tones are shifted to

$$f_{\text{tone}} = \frac{r}{N+k} \cdot f_s \cdot m, \qquad m = 1, 2, 3, \cdots \tag{3}$$

where r is the g.c.d. value of the DAC input code and the number of total unit elements $(N + k)$.

Regarding implementation cost, one extra unit element (i.e., $k = 1$) added to shift the tones near $f_s/2$ is most economic. Thus, for the nine-level DAC used in the previous section, the total unit elements become nine. Since code 4 is the most probable DAC input at small SDM input magnitude, according to (3), therefore, notable DAC tones are shifted to frequencies near $(f_s/9) \cdot m$ where m are positive integers. No obvious DAC tones close to $f_s/2$ and thus no obvious tones are aliased in the baseband. Simulation results confirm this analysis. SDM output spectra (with DWA, nine-unit elements, and input magnitude of -85 dB) is plotted in Fig. 3(a). The corresponding DAC noise spectra plotted in Fig. 3(b) shows that the notable DAC tones are moved to $(f_s/9) \cdot m$. In Fig. 3(c) and (d), SDM output and DAC noise spectra, with input magnitude of −45 dB, show no obvious baseband tones compared with Fig. 2(c) and (d). This technique significantly improves SNDR and in-band distortion (INBD), which is the total power of in-band tones above the noise level, as shown in Fig. 3(e) and (f), respectively.

Adding one extra unit element to shift the tones near $f_s/2$, one must ensure the shifted tones fall outside the baseband by satisfying the following inequality for the oversampling ratio (OSR):

$$\mathrm{OSR} > \frac{1}{2}(N+1). \tag{4}$$

Note that since the DAC tone spectra are located at multiple frequency bands centered at $1/(N+1)f_s \cdot m$, the oversampling ratio must be larger than the minimum value of $1/2(N+1)$ for safe design.

B. SDM's with Even Quantization Levels

For an $(N+1)$-level SDM where $(N+1)$ is an even number, using N unit elements for the DAC causes DAC notable tones near $f_s/2$ because, at small SDM input amplitudes, DAC input codes mostly concentrate at codes $(N-1)/2$ and $(N+1)/2$, these codes having a high probability of being interleaved with each other. Adding k extra unit elements can move the notable tones away from $f_s/2$, resulting in reduced baseband aliasing tone power. k extra unit elements added to the SDM's internal DAC does not change the quantization levels. At small SDM input magnitudes, notable DAC tones are shifted to

$$f_{\mathrm{tone}} = \frac{r}{2(N+k)} \cdot f_s \cdot m, \qquad m = 1, 2, 3, \cdots \tag{5}$$

where r is the g.c.d. value of the number of N and the number of total unit elements $(N+k)$. Only one extra unit element added is most economic. For the eight-level DAC used in the previous section, the number of unit elements becomes eight. DAC input sequence codes 3, 4, 3, 4, . . . , produce DAC noise as a periodic sequence, $e_0+e_1+e_2$, $e_3+e_4+e_5+e_6$, $e_7+e_0+e_1$, $e_2+e_3+e_4+e_5$, $e_6+e_7+e_0$, $e_1+e_2+e_3+e_4$, $e_5+e_6+e_7$, $e_0+e_1+e_2+e_3$, $e_4+e_5+e_6, \cdots$, resulting in the $f_s/16 \cdot m$ tone. Hence, tone power originally concentrated at $f_s/2$ is broken and distributed near $f_s/16 \cdot m$. Fig. 4(a)–(d) shows the SDM output and DAC noise spectra employing DWA with eight unit elements, with no notable tones aliased to the baseband. SNDR and INBD are significantly improved as shown in Fig. 4(e) and (f), respectively. From (5), it remains possible that the shifted tones fall in $fs/2$, causing baseband tones. However, numerous simulations show that adding one extra unit element to the DAC can greatly reduce notable tones near $f_s/2$ and in the baseband. Mathematical derivation shows that the minimum oversampling ratio is

$$\mathrm{OSR} > (N+1). \tag{6}$$

V. Performance Improvement

Monte Carlo simulation is used to analyze both component variation due to fabrication and also performance improvements resulting from utilization of the proposed technique.

Histograms of SNDR and INBD improvements via the presented technique are seen in Fig. 5(a) and (b), with one extra unit element in the nine-level DAC, a maximum random component mismatch of 0.5%, an input magnitude of −45 dB, an input frequency of $f_s/2048$, and 1000 trials. In the simulation, the component mismatch errors, $e_0, e_1, \cdots, e_8$ are randomly selected in the range between 0% to 0.5%. Because of component variability, mismatch error is variable. In the case of no extra DAC elements, the amount of resulting aliasing tone power ranges from approximately 0 dB to 30 dB above the noise level. In Fig. 5, SNDR and INBD improvements range from 0 to 30 dB, and DWA aliasing tone is nearly eliminated for all trials. Maximum component mismatches from 0.5% to 2% are also simulated. Averaging 1000 trials, SNDR and INBD improvements up to 12 and 20 dB are shown in Fig. 6(a) and (b), respectively, with SDM input magnitudes between −55 and −30 dB. Because of the occurrence of the occasional good component match, averaged error over a large number of trials does not adequately credit the serious mismatch cases. The main use of the proposed technique is to be seen in terms of increasing yield during mass production.

With one extra unit element in an eight-level DAC, histograms of SNDR and INBD improvements are shown in Fig. 5(c) and (d). Average SNDR and INBD improvements are up to 12 dB and 20 dB, as seen in Fig. 6(c) and (d), respectively.

Adding extra unit elements to different levels' DAC's for different SDM orders have also been simulated, confirming nearly perfect first-order DAC noise shaping with significant SNDR and INBD improvements and no notable SDM baseband tones, even at the higher levels of component variation.

VI. Conclusion

SDM aliasing tones related to DWA have been analyzed. Notable DAC tones are found closely correlated to the number of unit elements used in the DAC. An effective means for the suppression of aliased baseband tones has been proposed for the DWA algorithm. DAC mismatch errors due to fabrication are considered and Monte Carlo simulation is used to analyze this effect. Simulation confirms that incorporating the proposed technique with the DWA algorithm improves SNDR and INBD up to 12 dB and 20 dB on average, respectively, for 0.5% to 2% component variations, and with higher improvements for higher levels of component variation. It is suggested that this technique would be especially valuable for yield improvement in manufacturing situations, allowing more tolerant fabrication technique.

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