Dynamic Modeling and Control in Average Current Mode Controlled PWM DC/DC Converters

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Abstract– **In this paper, an improved averaging technique for average current mode controlled (ACMC) pulse width modulated (PWM) DC/DC converters is proposed. The modeling procedure provides ripple estimation and improves switching time calculation for ACMC PWM DC/DC converters. As an application of the theory, an ACMC boost converter with two proportional-integral (PI) controllers for both current loop and voltage loop is presented. Results confirm that the new averaged models are more accurate than conventional averaged models.**

I. Introduction

Performance of voltage-regulated supplies can often be improved by incorporating an inner current loop. This method, which is termed *current mode control* in PWM DC/DC converters, improves transient response, line regulation and simplifies the outer voltage loop controller design. In current mode control, the outer loop senses the output voltage and delivers a reference signal to the inner loop. The inner loop senses the inductor current (or switch current) and attempts to maintain either the maximum current value (*peak current mode control*) or the average current value (*average mode control*, Fig.1) constant during one switching cycle. Peak current mode control is the most popular technique, finding applications in PC power supplies, electric motor drives and consumer electronics. However, there are many applications in which the main problems in peak current mode control make average current mode control more desirable [4,5]. For example, peak current mode control allows a high distortion of the line current and results in unacceptable phase difference between input current and input voltage in a power factor correction circuit. Average current mode control has overcome these problems by having better line current phase disturbance rejection [5, 10].

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Dynamic modeling of power DC/DC converters is sometimes complicated due to the time-varying switching behavior of the diode and the transistor. Usually low order, nonlinear time-varying switching models can be produced by neglecting the dynamics occurred at frequencies much higher than the switching frequency. *Averaging* is then performed to obtain simplified approximate models which are time-invariant [2, 6, 7, 9, 14, 15] yielding *nonlinear averaged models.*

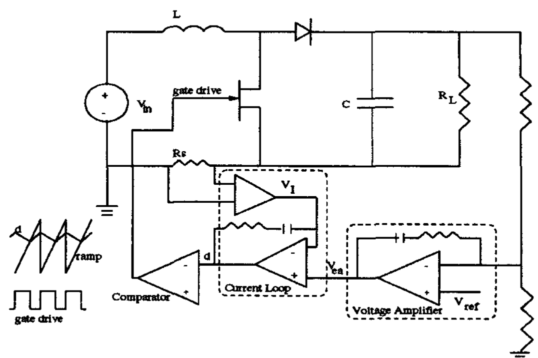

Figure 1: *A Typical ACMC Boost Converter*

Conventionally, as in [5, 17], the output of the control circuit is assumed to have linear relationship with the duty ratio. This is not always true in average current mode control topologies since the ripple in the fast current loop can not be neglected. In our research, this method is referred to be the "conventional averaged model".

Currently, there exists no general framework to obtain the control output ripple. As suggested in [4], this implies that ACMC converter models maybe inaccurate since they depend on the specific control topology. As a result, the designer is often left to analyze performance by a trial and error procedure using software such as PSpice.

The purpose of this paper is to develop an algorithm for the accurate modeling and control design for general topologies in average current mode control. The nonlinear averaged model derived in this paper is capable of describing the dynamics of both the power stage and the control

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circuitry such that the proper relationship between the duty ratio and the output of the control circuitry can be established.

The outline of this paper is as follows. Section II describes the limitation of conventional modeling methods for ACMC. Section III presents new averaged models for ACMC followed by experimental and numerical verification of the models in section IV. Section V gives conclusion.

II. Limitations of Conventional Averaging Techniques When Applied to ACMC Topologies

The elegance of averaging methods is that they provide methods to approximate the behavior of complicated time-varying switching systems with simpler time-invariant models. An underlying assumption of averaging in DC/DC converters is that the switching frequency is "sufficiently fast" [2, 9]. However, the conventional averaged model gives no insight or guidance as to "how fast" the switching frequency needs to be for acceptable closed loop performance. In fact, in a typical designed DC/DC converters ($25k\mathbf{H}_z$), Section *II.B* shows that conventional averaging is not accurate.

A. *Conventional Averaging Method*

General State Space Expression: As shown in Fig. 1, a PWM DC/DC converter usually is composed of two functional blocks: power stage and control circuits. Suppose that the DC/DC converter conducts continuously. The power stage has two topological modes due to the switching state: transistor on and transistor off. Then the general model of an open loop (power stage) DC/DC converter that includes the effects of internal resistances and other parasitics [1] can be given as:

$$\begin{aligned}\dot{\mathbf{x}} &= \mathbf{A}_0\mathbf{x}+\mathbf{B}_0\mathbf{V}_{in}+\mathbf{E}_0\mathbf{I}_0+\mathbf{G}_0+(\mathbf{A}_1\mathbf{x}\\ &+ \mathbf{B}_1\mathbf{V}_{in}+\mathbf{E}_1\mathbf{I}_0+\mathbf{G}_1)u(\mathbf{d}-\text{tri}(t,T))\\ \mathbf{y} &= \mathbf{C}_0\mathbf{x}+\mathbf{F}_0+\mathbf{H}_0\mathbf{I}_0+(\mathbf{C}_1\mathbf{x}+\mathbf{H}_1\mathbf{I}_0\\ &+ \mathbf{F}_1)u(\mathbf{d}-\text{tri}(t,T))\end{aligned} \quad (1)$$

where $\mathbf{x}\in\mathbf{R}^n$ is the state vector; $\mathbf{y}\in\mathbf{R}$ is the output of the system. $\mathbf{V}_{in}\in\mathbf{R}$ is the input source voltage and $\mathbf{I}_o$ is the disturbance load current. u is the heaviside step function, i.e. $u(s)=1$ when $s\geq 0$ and $u(s)=0$ when $s<0$. $0\leq\mathbf{d}\leq 1$ is the duty ratio and $\text{tri}(t,T)=\frac{t \bmod T}{T}$.

In average current mode control, both the output voltage and the inductor current are the feedback variables. Thus, let $\mathbf{x}_C$ denotes the control state vector and $\mathbf{d}$ be the output of the control circuitry. Then we have

$$\begin{aligned}\dot{\mathbf{x}}_C &= \mathbf{A}_C\mathbf{x}_C+\mathbf{B}_C\begin{bmatrix}V_{ref}-\mathbf{V}_o\\ -\mathbf{V}_I\end{bmatrix}+\mathbf{C}_C V_{ref}\\ \mathbf{d} &= \mathbf{G}_C\mathbf{x}_C+\mathbf{K}_C\begin{bmatrix}V_{ref}-\mathbf{V}_o\\ -\mathbf{V}_I\end{bmatrix}+\mathbf{E}_C V_{ref}\end{aligned} \quad (2)$$

where $\mathbf{V}_I$ is proportional to the inductor current $\mathbf{i}_L$ and can be written as $\mathbf{V}_I=\mathbf{A}_{GII}\mathbf{i}_L$. $\mathbf{V}_{ref}$ is the reference voltage for outer voltage loop. $\mathbf{d}$, which also appears in (1) and is a function of both power stage and control circuitry state variables, represents the output of the control circuit.

Conventional Model: For sufficiently small switching period, [2, 9] show that $\mathbf{x}(t)\approx\bar{\mathbf{x}}(t)$ where $\bar{\mathbf{x}}(t)$ is given as the solution to the "averaged equation":

$$\begin{aligned}\dot{\bar{\mathbf{x}}} &= \mathbf{A}_0\bar{\mathbf{x}}+\mathbf{B}_0\mathbf{V}_{in}+\mathbf{E}_0\mathbf{I}_0+\mathbf{G}_0+(\mathbf{A}_1\bar{\mathbf{x}}\\ &+ \mathbf{B}_1\mathbf{V}_{in}+\mathbf{E}_1\mathbf{I}_0+\mathbf{G}_1)\bar{\mathbf{d}}\end{aligned} \quad (3)$$

This, of course, is the classical averaged model given by [3, 7, 14, 15]. Similarly, [15] gives the averaged output equation as:

$$\mathbf{w}=\mathbf{C}_0\bar{\mathbf{x}}+\mathbf{F}_0+\mathbf{H}_0\mathbf{I}_0+(\mathbf{C}_1\bar{\mathbf{x}}+\mathbf{H}_1\mathbf{I}_0+\mathbf{F}_1)\bar{\mathbf{d}} \quad (4)$$

In the above power stage averaged models, $\bar{\mathbf{d}}$ is assumed to be the output of the control circuitry with the averaged state variables and output voltage of the power stage as the inputs. Thus, in average current mode control topology, averaged models of the control circuitry are given as:

$$\begin{aligned}\dot{\bar{\mathbf{x}}}_C &= \mathbf{A}_C\bar{\mathbf{x}}_C+\mathbf{B}_C\begin{bmatrix}\mathbf{V}_{ref}-\bar{\mathbf{V}}_O\\ -\bar{\mathbf{V}}_I\end{bmatrix}+\mathbf{C}_C\mathbf{V}_{ref}\\ \bar{\mathbf{d}} &= \mathbf{G}_C\bar{\mathbf{x}}_C+\mathbf{K}_C\begin{bmatrix}\mathbf{V}_{ref}-\bar{\mathbf{V}}_O\\ -\bar{\mathbf{V}}_I\end{bmatrix}+\mathbf{E}_C\mathbf{V}_{ref}\end{aligned} \quad (5)$$

where the only change from (2) is in taking $\overline{\mathbf{V}}_I$ and $\mathbf{w}$ instead of $\mathbf{V}_I$ and $\mathbf{y}$.

B. *Explanation of Conventional Modeling Errors*

It has been documented that conventional averaging techniques have limitations in modeling two loop control PWM DC/DC converters [1], such as DC offset of the output voltage and incapability of predicting the instability of the closed loop. Accurate modeling of such systems still remains an open area of research.

In average current mode control topologies, there is large flexibility in selecting the control schemes for both the inner current loop and the outer voltage loop. With each different control scheme, the deficiencies of the conventional averaged models will vary. However, in common, the conventional averaged models can not accurately track the transient response of the DC/DC converters.

Fig. 2 plots the start-up output voltage of the average current mode controlled boost converter with two PI controllers for the two loops. (Section V presents the details of the circuit topology.) The conventional averaged models do not accurately model the average output voltage when the system is in transient. As $t \to \infty$ the PI controllers eventfully force the steady state output voltage of the converter to be the required values as indicated by the conventional averaged models. Therefore, for this circuit topology, inaccuracies are in modeling transient response and not steady state DC-offset (for other topologies, this might not be true).

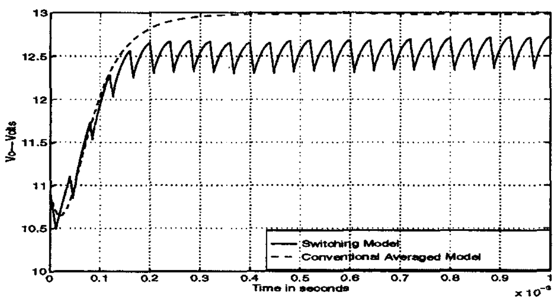

Figure 2: Output Voltage of a ACMC Boost Converter with two PI controllers

The reason for the inaccuracies of the conventional models is that the output of the control circuitry is no longer ripple negligible, as shown in Fig. 3. If the conventional averaged models were accurate, the dashed curve in Fig. 3 would track the averaged value of the switching curve. It does not, and as a result, the conventional model incorrectly predicts the transistor/diode switching time when the system is in transient.

These results motivate the need for more accurate modeling in average current mode control.

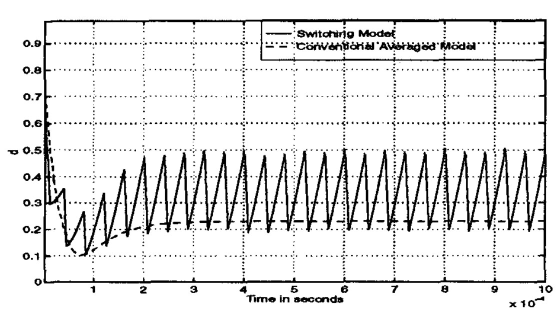

Figure 3: Output of the Control Circuitry

III. Switching Frequency Dependent Model

In this section, we make the necessary extensions of [1] to generalize the switching frequency dependent averaged models to average current mode control topologies, which have dynamic feedback controllers.

The difficulty in obtaining accurate averaged models for two loop PWM DC/DC converters is that there is a ripple in the signal sent to the comparator, given as $\mathbf{d}$ in (2). As a result, there has been no general procedure which can accurately predict when $\mathbf{d}(t) = \text{tri}(t, T)$, i.e. the switching time of the transistor. The following step-by-step procedure begins to solve this open problem.

A. *Step 1: Power Stage Averaged Equations*

Let $\tau_s T$ denotes the instant before $u(\mathbf{d} - \text{tri}(t,T))$ switches from 1 to 0. The variable τ_s represents the "true" duty ratio and is not necessarily equal to $\mathbf{d}$ as suggested by conventional technique [1]. Then we propose that the switching frequency dependent averaged model for the power stage is :

$$\begin{aligned}\dot{\bar{\mathbf{x}}} &= \mathbf{A}_0\bar{\mathbf{x}} + \mathbf{B}_0\mathbf{V}_{in} + \mathbf{E}_0\mathbf{i}_0 + \mathbf{G}_0 + (\mathbf{A}_1\bar{\mathbf{x}} \\ &+ \mathbf{B}_1\mathbf{V}_{in} + \mathbf{E}_1\mathbf{I}_0 + \mathbf{G}_1)\tau_s \\ \mathbf{w} &= (\mathbf{C}_0\bar{\mathbf{x}} + \mathbf{F}_0) + \mathbf{H}_0\mathbf{I}_0 + (\mathbf{C}_1\bar{\mathbf{x}} + \mathbf{H}_1\mathbf{I}_0 + \mathbf{F}_1)\tau_s \quad (6)\end{aligned}$$

where all matrices are as in (1), $\bar{\mathbf{x}} \in \mathbf{R}^n$ is the averaged state vector and $\mathbf{w}$ is the averaged output of (1). The difficult part of the analysis is to determine τ_s, which, at present, is still unknown and will depend on the switching frequency.

B. *Step 2: Power Stage Ripple*

The work of [1] suggests that the power stage state vector can be approximated by $\mathbf{x} \approx \bar{\mathbf{x}} + \Psi_x$ (Fig. 4), where Ψ_x is the zero average state ripple and given in [1].

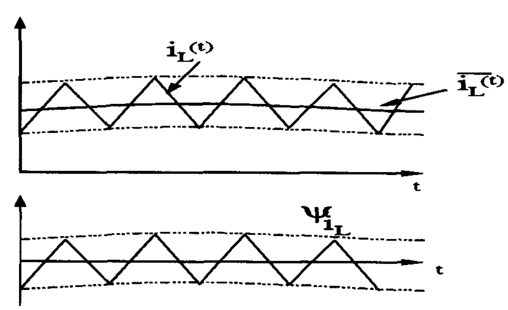

Figure 4: *Typical Ripple on State Variable: Inductor Current*

Likewise, we generalize the results of [14] (by replacing $\mathbf{d}$ by τ_s) to closed loop systems and write $\mathbf{y} = \mathbf{w} + \Psi_{V_o}$ (Fig.

5), where Ψ_{V_o} is the ripple on the output voltage. Using the estimate that $\mathbf{x} \approx \bar{\mathbf{x}} + \Psi_x$, in $\Psi_{V_o} = \mathbf{y} - \mathbf{w}$, and using (1) and Ψ_x yields,

$$\Psi_{Vo} = \begin{bmatrix} (\mathbf{C}_1\mathbf{z} + \mathbf{H}_1\mathbf{I}_o + \mathbf{F}_1)(1-\tau_s) + (\mathbf{C}_0 + \mathbf{C}_1)\Psi_x, \\ 0 \le t\text{mod}T \le \tau_s T \\ -(\mathbf{C}_1\mathbf{z} + \mathbf{H}_1\mathbf{I}_o + \mathbf{F}_1)\tau_s + \mathbf{C}_0\Psi_x, \\ \tau_s T \le t\text{mod}T \le T \end{bmatrix}$$

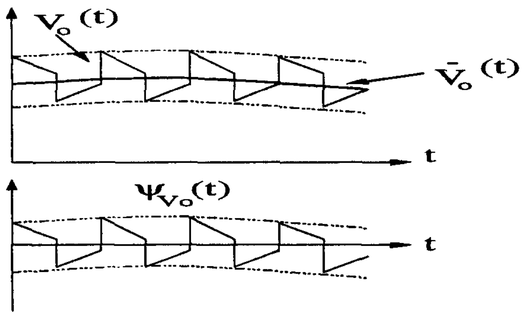

Figure 5: *Typical Ripple on the Output Voltage for a Boost Converter with High ESR*

C. *Step 3: Superposition Principle for Control Circuitry*

General control loops of linearly controlled average current mode control DC/DC converters can be drawn in block diagram as in Fig. 6.

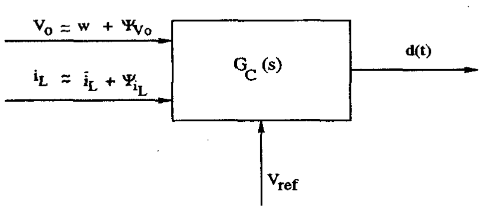

Figure 6: *Control Circuitry*

Since the controller is linear, we propose to apply superposition as in Fig. 7. Both the inductor current and output voltage are fed into the controller as inputs. The output, $\mathbf{d}(t)$, is a scalar.

As previously derived, each of the inputs can be written as the sum of an averaged value state variable plus a zero average ripple. Therefore, using (2) and superposition, we have control equations:

$$\begin{aligned} \dot{\bar{\mathbf{x}}}_C &= \mathbf{A}_C\bar{\mathbf{x}}_C + \mathbf{B}_C\begin{bmatrix} \mathbf{V}_{ref} - \bar{\mathbf{V}}_o \\ -\mathbf{A}_{GII}\bar{\mathbf{i}}_L \end{bmatrix} + \mathbf{C}_C\mathbf{V}_{ref} \\ \mathbf{d}_1 &= \mathbf{G}_C\bar{\mathbf{x}}_C + \mathbf{K}_C\begin{bmatrix} \mathbf{V}_{ref} - \bar{\mathbf{V}}_o \\ -\mathbf{A}_{GII}\bar{\mathbf{i}}_L \end{bmatrix} + \mathbf{E}_C\mathbf{V}_{ref} \quad (7) \end{aligned}$$

and

$$\begin{aligned} \dot{\Psi}_C &= \mathbf{A}_C\Psi_C + \mathbf{B}_C\begin{bmatrix} -\Psi_{Vo} \\ -\mathbf{A}_{GII}\Psi_{i_L} \end{bmatrix} \\ \mathbf{d}_2 &= \mathbf{G}_C\Psi_C + \mathbf{K}_C\begin{bmatrix} \Psi_{V_o} \\ -\mathbf{A}_{GII}\Psi_{i_L} \end{bmatrix} \quad (8) \end{aligned}$$

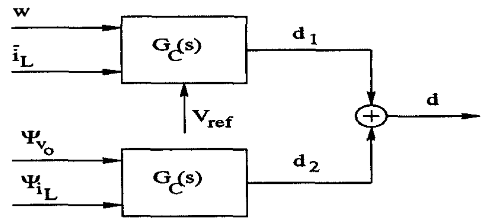

Figure 7: *Superposition Model of Control Circuitry*

Notice that (8) is time-invariant and there is no need to average it. If we assume that Ψ_C has zero initial conditions at the beginning of each switching period, then

$$\Psi_C = -\mathbf{B}_C\int_0^t e^{\mathbf{A}_C(t-\tau)}\begin{bmatrix} -\Psi_{Vo} \\ -\mathbf{A}_{GII}\Psi_{I_L} \end{bmatrix} d\tau \quad (9)$$

which gives the general expression of the ripple on the control output.

D. *Step 4: Switching Time*

In the above steps, it was assumed that τ_s was a known function of the state, although it is not. The switching time $\tau_s T$ is defined as the instant before the output of the control circuitry, $\mathbf{d}(t)$, is intersected with the external ramp which is normalized to be a DC offset of 0 and peak to peak value of 1. Specifically, at $\frac{t\text{mod}T}{T} = \tau_s T$, we have $\mathbf{d}(\tau_s T) = \tau_s$. This implies:

$$\mathbf{d}_1 + \mathbf{G}_C\Psi_C(\tau_s T) + \mathbf{K}_C\begin{bmatrix} -\Psi_{Vo}(\tau_s T) \\ -\mathbf{A}_{GII}\Psi_{i_L}(\tau_s T) \end{bmatrix} = \tau_s \quad (10)$$

which yields a nonlinear algebraic equation to solve for τ_s, $0 \le \tau_s \le 1$. Notice that as $T \to 0$, $\tau_s \to \mathbf{d}_1$, which yields the conventional averaged models, as expected. Equations (3), (5) and (8) together represent the newly proposed averaged models. We remark that (8) will depend on T, the switching period. Therefore, the averaged models are switching frequency dependent and include the effects of ripple on control signals (not modeled in conventional techniques).

IV. Simulations and Experimentation

In this paper, we present the PI control topology as an example. Consider Fig.1 with $\mathbf{V}_{in} = 10\mathbf{V}$(input voltage),

$L = 200.9\mu\mathbf{H}$, $C = 21.87\mu\mathbf{F}$, $R_C = 0.167\Omega$(ESR), and $R_L = 13.2\Omega$(Load resistor),

Then, in the power stage, the state variables are inductor current $x_1 = \mathbf{i}_L$ and capacitor voltage $x_2 = \mathbf{v}_C$, and the output is set to be the output voltage $y = \mathbf{V}_o$. In this case:

$$\alpha = \frac{R_L}{R_L + R_C}, \mathbf{A}_0 = \begin{bmatrix} -\frac{\alpha R_C}{L} & -\frac{\alpha}{L} \\ \frac{\alpha}{C} & -\frac{1}{C(R_L+R_C)} \end{bmatrix},$$
$$\mathbf{A}_1 = \begin{bmatrix} \frac{\alpha R_C}{L} & \frac{\alpha}{L} \\ \frac{\alpha}{C} & 0 \end{bmatrix}, \mathbf{B}_1 = \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix}, \mathbf{B}_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix},$$
$$\mathbf{C}_0 = \begin{bmatrix} \alpha R_C & \alpha \end{bmatrix}, \mathbf{C}_1 = \begin{bmatrix} -\alpha R_C & 0 \end{bmatrix}, \quad (11)$$
$$\mathbf{F}_0 = \mathbf{F}_1 = 0, \mathbf{H}_0 = \alpha R_C, \mathbf{H}_1 = -2\alpha R_C.$$
$$\mathbf{E}_0 = \begin{bmatrix} -\frac{\alpha R_C}{L} & \frac{\alpha}{C} \end{bmatrix}^T, \mathbf{E}_1 = \begin{bmatrix} \frac{\alpha R_C}{LC} & -\frac{2\alpha}{C} \end{bmatrix}^T$$

The control circuitry are designed according to small signal models to avoid subharmonic oscillation, guarantee the relative stability and closed loop performance. The control parameters for the control topologies are: $\mathbf{A}_{GII} = 10$,$k_{PI} = 0.1$, $k_{II} = 10.3$, $k_{PV} = 0.01$ and $k_{IV} = 10.4$, and the model is given by (2) with (Fig. 8):

$$\mathbf{A}_C = \begin{bmatrix} 0 & 0 \\ \beta k_{II} & 0 \end{bmatrix}, \mathbf{B}_C = \begin{bmatrix} k_{IV} & 0 \\ \beta k_{PV} k_{II} & k_{II} \end{bmatrix},$$
$$\mathbf{C}_C = \begin{bmatrix} (\beta - 1)k_{IV} & \frac{\beta k_{II}}{\beta - 1}(k_{PI} + \beta - 1) \end{bmatrix}^T,$$
$$\mathbf{G}_C = \begin{bmatrix} 1 + \beta k_{PI} & 1 \end{bmatrix}, \mathbf{E}_C = (1 + \frac{k_{PV}}{\beta - 1})(1 + \beta k_{PI}),$$
$$\mathbf{K}_C = \begin{bmatrix} k_{PV}(1 + \beta k_{PI}) & k_{IV} \end{bmatrix} \quad (12)$$

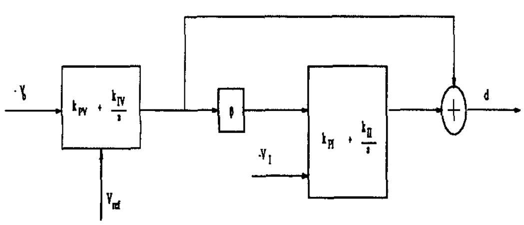

Figure 8: Two PI Control Topology

where β is applied to balance the reference voltage for the outer loop. Using the formula derived in step 2, leads to:

$$\Psi_x(\tau_s T) \approx \frac{T\tau_s(1-\tau_s)}{2} \begin{bmatrix} \frac{\alpha}{L}(R_C \mathbf{z}_1 + \mathbf{z}_2) \\ -\frac{\alpha}{C}\mathbf{z}_1 \end{bmatrix} \quad (13)$$
$$\Psi_{Vo}(\tau_s T) \approx -\alpha R_C \mathbf{z}_1(1-\tau_s) - \frac{T\alpha^2 \mathbf{z}_1 \tau_s(1-\tau_s)}{2C}$$

where $\mathbf{z}_1$ and $\mathbf{z}_2$ are the averaged state variables of $\mathbf{i}_L$ and $\mathbf{v}_C$ respectively. Omitting extensive algebraic details, we derive

$$\Psi_C(\tau_s T) \approx \begin{bmatrix} -k_{IV} T\alpha R_C \mathbf{z}_1(\tau_s^2 - 1) \\ \frac{(\tau_s^3 - \tau_s^2)}{12}\beta k_{II} k_{IV} \alpha \mathbf{z}_1(\frac{\alpha\tau_s T}{C} - \\ 6R_C) + k_{PV} k_{II} \beta\alpha R_C \mathbf{z}_1(\tau_s - \tau_s^2)T \end{bmatrix} \quad (14)$$

Now substituting (13) and (14) into (10), the switching time τ_s can be obtained. Together with (6), this yields a more accurate circuit approximation. Clearly, the novelty of the approach is its ability to make a distinction between the average value of $\mathbf{d}$ and the switching time. In conventional averaging techniques, these two quantities are forced to be equal.

A. *Start Up Simulations*

The simulation results for start up of the boost converter with PI control topology are recorded in Fig. 9 (output voltage). From these figures, we can see that in this control topology, the conventional averaged models do not reflect the dynamics of the switching system, reaching their steady state values too quick. The switching frequency dependent models, on the other hand, track the transient states of the switching system. The simulation results also show that the new models are able to predict the correct switching time of the transistor ($\tau_s T$) as well as the averaged value of the output of the control circuitry ($\mathbf{d_1}$)(Fig. 10). The conventional models, on the other hand only predict the correct steady state switching time. The fact that this switching time is not the true average value of the controller output leads to the inaccuracies in the transient response.

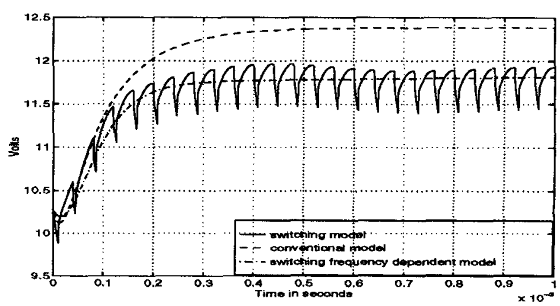

Figure 9: *Output Voltage of Start Up*

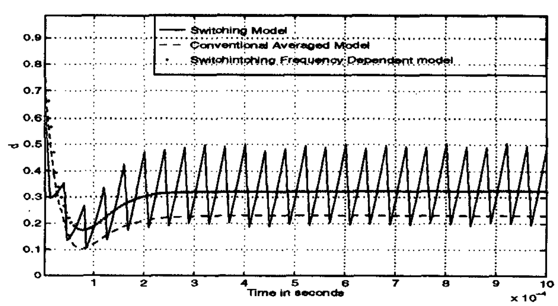

Figure 10: Output of the Control Circuitry

B. *Load Variation Simulations and Experimentation*

The load resistance is suddenly changed from 13.2Ω to 26.4Ω. Fig.11 recorded the experimentation result and the theoretical estimations. It can be seen that both conventional model and the switching frequency dependent model can reflect the tendency of the power stage. But the conventional model gives the rise of the over-adjustment, i.e, predicting a peak voltage of 16.2V instead of the true voltage of 15.5V (predicted by the new model) immediately after the load variation.

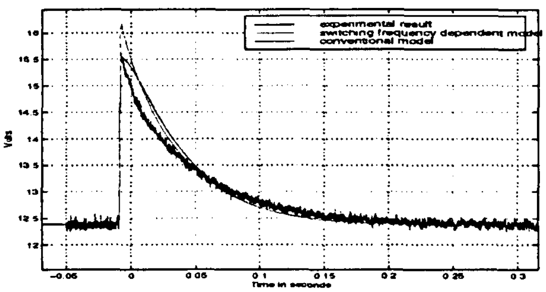

Figure 11: *Output Voltage of Load Variation*

V. Conclusion

ACMC PWM DC/DC converters have the potential to take various control schemes for their current and voltage loops, which makes the closed loop modeling difficult and subjective to inaccuracy. Conventional averaging methods do not reflect the dynamics of the control circuitry, leading to inaccuracies in the prediction of the averaged output voltage and the averaged duty ratio. The averaging technique proposed in this paper integrates the dynamics of the power stage with those of dynamic control circuitry and generates accurate averaged models which track the dynamics of the closed loop system.

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