Two-Pole Compensation

Part 2: Generalized Two-Pole Compensator

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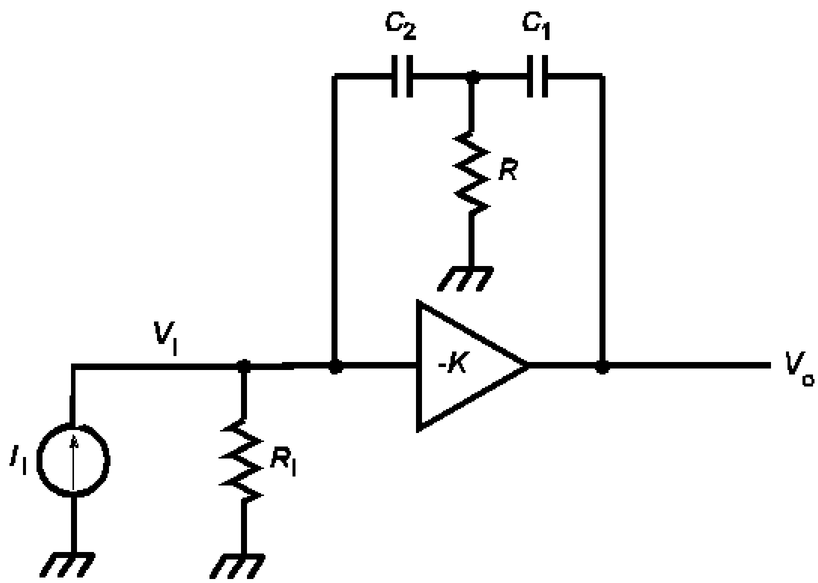
The previous development in Part 1 http://www.analogZONE.com/col_0628.htm assumed real and equal poles for the two-pole compensator. What happens if the circuit is modified to allow for complex poles? The benefit in doing this, if it can be done, is that for amplifiers with other poles and zeros in the loop gain, complex pole-pair compensation can be achieved by the compensator while maintaining high loop gain over bandwidth.

Two-Pole Compensator Circuit Design

The two-pole compensator circuit of part 1 is reproduced below.



The design equations of Part 1 constrained the damping ratio, ζ, of the pole-pair to unity. ($\zeta = \cos(\phi)$, where ϕ is the quadratic pole angle.) A useful design parameter, the pole-zero separation, will be given its own symbol as defined:

$$\frac{z}{\|p\|} = \frac{z}{\omega_n} = \gamma$$

Noting that the location of the zero, $z = 1/\tau_z$, and that the quadratic pole is of the form:

$$as^2 + bs + 1 = \left(\frac{s}{\omega_n}\right)^2 + \left(\frac{2\zeta}{\omega_n}\right) \cdot s + 1,$$

then z can be expressed in terms of design parameters as follows:

$$z = \gamma\omega_n \Rightarrow \frac{1}{\omega_n} = \gamma\tau_z \Rightarrow a = \gamma^2\tau_z^2 \Rightarrow (K+1)RR_iC_1C_2 = \gamma^2R^2(C_1+C_2)^2$$

Where, a is taken from the voltage gain of the compensator circuit (see part 1.) Solving for R:

$$R = R_i \cdot \frac{(K+1)(C_1 \| C_2)}{\gamma^2(C_1+C_2)}$$

Now ζ is brought in as,

$$\zeta = \frac{b}{2\sqrt{a}} = \frac{b}{2} \cdot \omega_n = \frac{\tau_z + R_i C_2}{2} \cdot \omega_n$$

The compensator element C_2 value results from solving this equation, and is:

$$C_2 = \frac{(2\zeta\gamma - 1)\tau_z}{R_i}, \qquad \zeta > \frac{1}{2\gamma}$$

C_2 is expressed entirely in given parameters and is thereby determined. Next, the equation for z:

$$z = \frac{1}{R(C_1 + C_2)} = \frac{1}{\tau_z},$$

is solved for C_1:

$$C_1 = \frac{\tau_z}{R} - C_2$$

and substituted into the previous equation for R. This results in an expression for R in given parameters and C_2, which is known from the equation above:

$$R = \frac{\tau_z}{C_2}\left[1 - \left(\frac{\gamma^2}{K+1}\right) \cdot \left(\frac{\tau_z}{R_i C_2}\right)\right], \qquad R_i C_2 > \left(\frac{\gamma^2}{K+1}\right) \cdot \tau_z$$

Finally, R is now substituted into the equation for C_1 to yield:

$$C_1 = C_2\left[\frac{1}{1 - [\gamma^2/(K+1)][\tau_z / R_i C_2]} - 1\right]$$

Two-Pole Compensator Design Example

An amplifier has a gain of $K = 10$ k, $R_i = 10$ kΩ, is to be two-pole compensated to have a zero at 500 kHz, and begin its rolloff a decade lower, at 50 kHz. Furthermore, an MFED pole response (30° pole angle) is desired, where $\zeta \cong 0.866$. Component tolerances are 5%.

The required parameters are:

$$\tau_z = \frac{1}{2\pi f_z} = \frac{1}{2\pi(500\ \text{kHz})} = 318\ \text{ns}$$

$$\gamma = \frac{z}{\omega_n} = \frac{500\ \text{kHz}}{50\ \text{kHz}} = 10$$

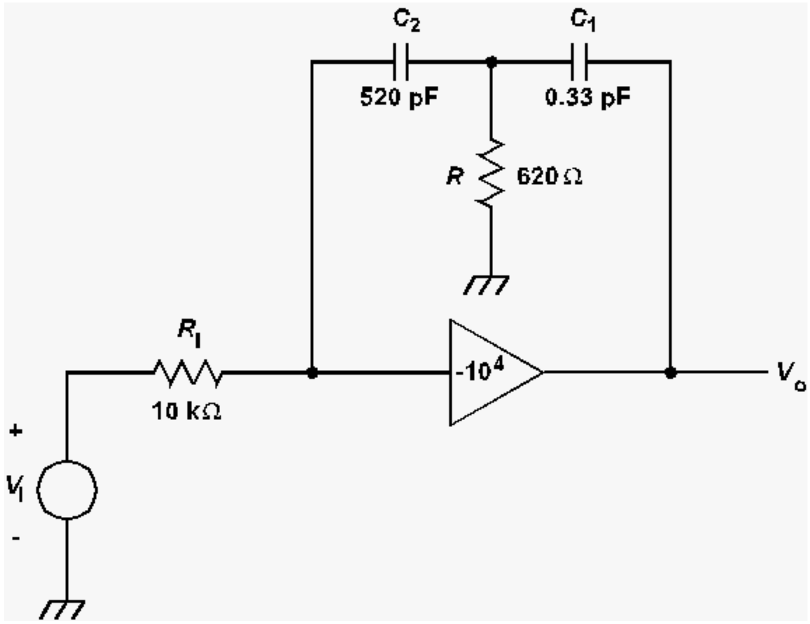
First we calculate C_2; it is 519 pF. The closest 5% part is $C_2 = 520\ \text{pF}$

Next, calculate R using the calculated value for C_2 (instead of the 5% value) to keep the calculations accurate. (This is important when we get to C_1 because the difference of two large numbers is taken.) Then $R = 613\ \Omega$. The closest value is $R = 620\ \Omega$

Finally, C_1 is calculated from its equation, or from:

$$C_1 = \frac{\tau_z}{R} - C_2,$$

and if care is taken to retain numerical consistency it is 0.32 pF. This is a very small discrete capacitor value and suggests that it might be difficult to realize this reliably as a discrete circuit in manufacturing because this value is on the order of parasitics. The circuit-board layout between the output node and R must minimize stray capacitance. One way to implement C_1 is with a small trimmer capacitor of about 1 pF maximum value. If such a small C_1 is not feasible, then the given parameters must be adjusted to result in a larger capacitance. C_1 increases if R increases due to a decrease in C_2. And C_2 decreases when ζ, γ, or τ_z decrease or R_i increases. The amplifier design is shown below.



To check these results, we turn from synthesis to analysis and calculate a and b of the pole factor:

$$a = RR_iC_1C_2(K+1) = 1.06\times10^{-11}\,\mathrm{s}^2 \rightarrow f_n = 48.8\,\mathrm{kHz} \cong 50\,\mathrm{kHz}$$

$$b = R(C_1 + C_2) + R_iC_2 = 318\,\mathrm{ns} + 5.19\,\mu\mathrm{s} = 5.51\ \mu\mathrm{s} \Rightarrow \zeta = 0.85 \cong 0.87$$

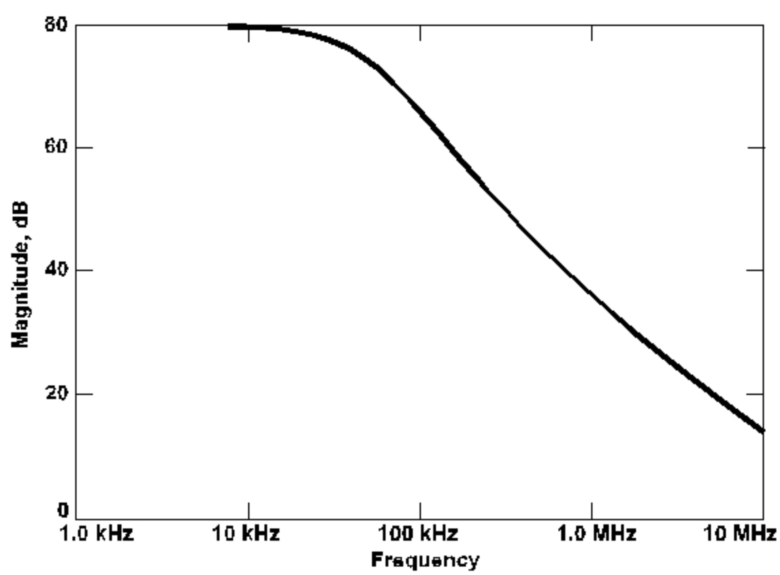
Both f_n and ζ are within the 5% tolerance of the components. Finally, we check our results against the constraints:

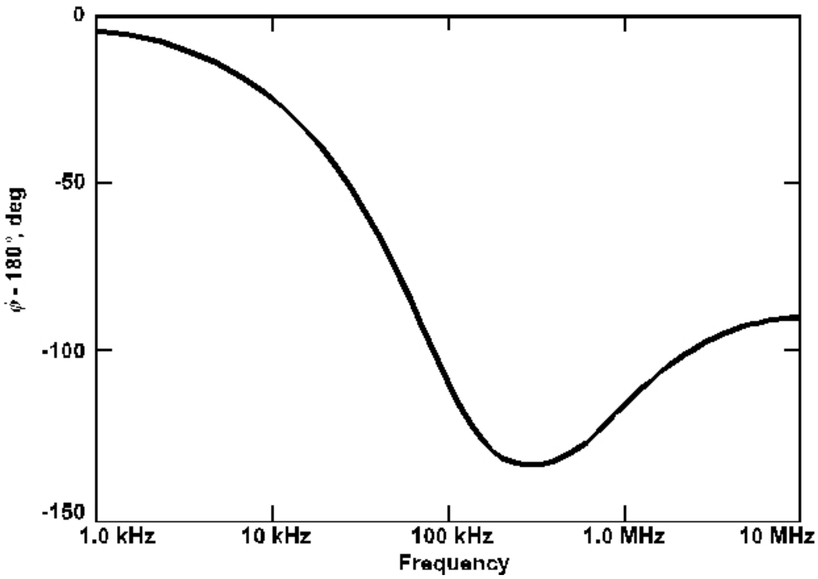
$$\zeta = 0.87 > \frac{1}{2\gamma} = \frac{1}{20} = 0.05 \qquad \text{(checks)}$$

$$C_2 = 0.33\,\mathrm{pF} > \frac{\gamma^2\tau_z}{R_i(K+1)} = 0.32\,\mathrm{pF} \quad \text{(checks)}$$

The lower limit of C_2 is approached because C_1 and C_2 are so widely separated.

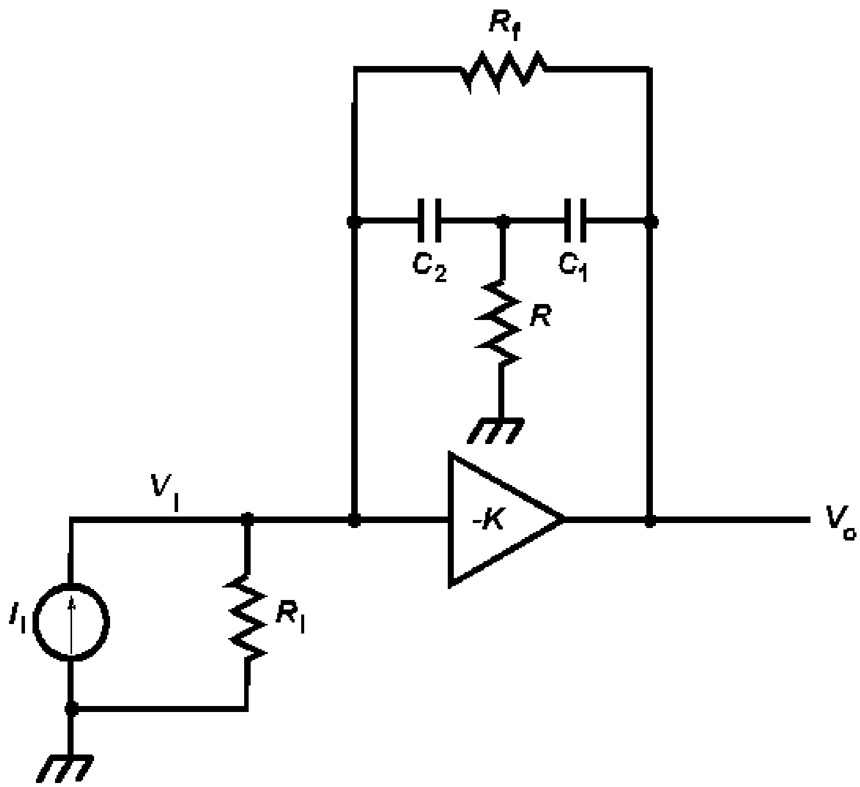
The final check of this example is made from the SPICE frequency-response simulation. The Bode plots are shown below.





Can the Compensator Circuit be Statically Stabilized?

The above amplifier has no static (dc) feedback and behaves like an integrator at 0 Hz (dc). Unless it is within a larger feedback loop, the output drifts out of its linear range due to offset errors. For stand-alone applications, R_f must be included for static stabilization, as shown below. (The resulting compensator has the topology of a *bridge-T* filter.)



The transfer function, with R_f included, is approximately the same as before (in Part 1) under the conditions that:

$$R_f >> \begin{cases} R_i \\ 1/sC_2 + (1/sC_1 \parallel R) \end{cases}$$

Under these conditions, the static-path feedback through R_f is small compared to the capacitive path (yet enough to statically stabilize the amplifier), and R_f negligibly shunts R_i and does not affect the transfer function of the capacitive path. Two-pole compensation can be achieved with limited, but often adequate, static feedback and all the theory developed thus far can be applied.

Two-Pole Op-Amp Circuit?

What happens if an op-amp is used? The voltage gain for $K \to \infty$ is:

$$\left.\frac{V_o}{V_i}\right|_{K\to\infty} = -\frac{R_f}{R_i} \cdot \frac{sR(C_1 + C_2) + 1}{s^2 R_f RC_1C_2 + sR(C_1 + C_2) + 1}$$

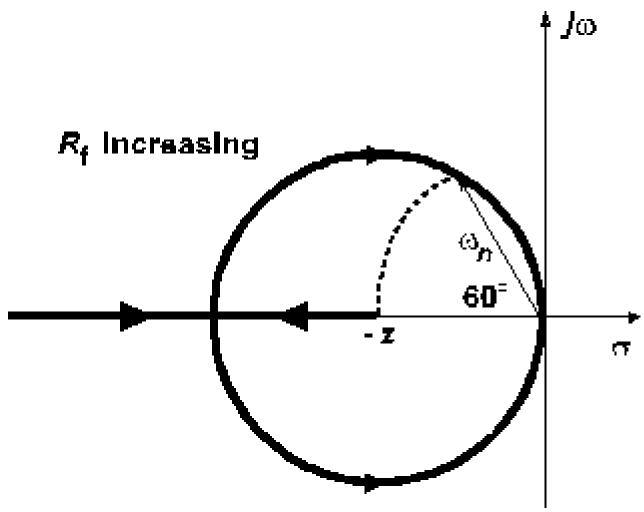
As $R_f \to \infty$, the closed-loop voltage gain approaches the same expression as in Part 1, as it must. The above gain differs in that $b = \tau_z$ and does not have the extra degree of freedom that the previous circuit does with its R_iC_2 term. Consequently, ζ and γ are not independent but are related by:

$$\zeta = \frac{b}{2}\omega_n = \frac{\tau_z \omega_n}{2} \Rightarrow \zeta = \frac{1}{2\gamma} \Rightarrow \gamma = \frac{1}{2\zeta}$$

Proceeding as in the prior derivation:

$$R = \frac{\tau_z}{C_2}\left(1 - \frac{\tau_z}{\gamma^2 R_f C_2}\right), \qquad C_2 > \frac{\tau_z}{\gamma^2 R_f}$$

C_2 is chosen to satisfy the constraint that $R > 0$. This choice depends upon R_f and interacts with it. The pole locus of voltage gain of the previous circuit was varied by $(K + 1)$ since it was in a but not b. For this compensator, variation with constant b is due to R_f instead. To achieve $\gamma > 1$, as required for a two-pole compensator, the poles must be complex and have a pole angle greater than 60°, as shown below.



At a pole-zero separation of zero, a 60° pole angle results, which establishes the minimum frequency peaking, M_m as 1.15 (or 1.25 dB) and minimum pulse overshoot fraction, M_p as 16 %. For a somewhat useful compensator with one octave of pole-zero separation, $\gamma = 2$, and $\zeta = 0.25$ ($\phi = 76°$), $M_m = 2.97$ (or 6.3 dB), and $M_p = 44$ %.

We saw in Part 1 that operational amplifier forward paths drove the circuit poles to the origin, defeating the two-pole scheme. With non-large R_f, the poles again become finite but, because of the unavoidable underdamped response that accompanies adequate pole-zero separation, the bridge-T two-pole op amp compensator is very limited for two-pole compensation. It functions better as a notch filter, which is a typical application for bridge-T networks.

Conclusion

The design equations and a design example of the two-pole compensator circuit have been presented. With the math worked out, use of this design procedure is not difficult and can result in better feedback amplifier accuracy and linearity at higher frequencies than dominant-single-pole compensation. Remember, two-pole compensation is not used to increase amplifier stability but to increase upper-frequency loop gain. Two-pole compensation tends to decrease stability and must be applied carefully, making sure that no uncompensated poles exist in the loop below the two-pole break frequency.

Secondly, this compensation technique, when implemented using the given circuit, is best placed within a larger feedback loop or else static errors will cause it to drift out of range. This problem can usually be corrected by simply placing a large-value feedback resistor around the finite-gain amplifier. However, if an op amp is used, the pole-zero placement for two-pole compensation is constrained excessively, rendering the attempt a failure. Not every "good idea" results in something useful.

Much of the content of this article was derived from *Analog Circuit Design*, volume 2: *Dynamic Circuit Response*, available at http://www.innovatia.com

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